Structured H_∞ Synthesis Method with Interval Analysis : Application to the Robust Control of an AUV

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I. Introduction

This paper presents a robust and structured controller synthesis method dealing with uncertain system dynamics. Postulating time-invariant and bounded errors on both system inner parameters and external disturbances this method provides formal proof of compliance to a given set of stability and performance criteria. This guarantee is obtained through the interval analysis of the closed-loop system which encompasses any possible time-domain and frequency-domain variations. Here this method is applied to the robust frequency attenuation, seen as H_∞ criteria, and stability for depth control of a Autonomous Underwater Vehicle (AUV), the Iver2. It emphasizes the synthesis method from model and regulation problem definitions to synthesis implementation and discuss its results.

II. Equipments and Goals

A. The Iver2 AUV

The Iver2 is an Autonomous Underwater Vehicle (AUV) designed by OceanServer. Its main characteristics are its ability to be operated by a single person and the large variety of payloads that it can sustain. It results in its great ability to carry out very different missions like bathymetry or mapping and localization. Its modularity pushes for the development of a synthesis method able to provide a reliable controller that can offer performance guarantees for a large set of payload configurations.

B. IROS Dynamic Model

Here one suggests a dynamic model for the Iver2's depth with explicit parameter uncertainties. [1] provides a complete dynamic model of the EchoMapper version of the Iver2 which is a specific payload configuration. It is based on the 6 degree-of-freedom standard equations described by Fossen in [2]. Model frames and states are illustrated 1 with $\nu = (u, v, w, p, q, r)$ be the speeds and rotation speeds around axis of the body-fixed frame.

Several hypothesis can be set in order to simplify the dynamic model. Due to low coupling in the model directions, it is possible to consider that both the depth and pitch do not interfere with dynamics along other axis. Obviously, depth behavior is highly correlated to the pitch. It allows us to only use the two non-linear equations related to w and q to describe the depth dynamic, as speed, heading and roll are assumed steady.

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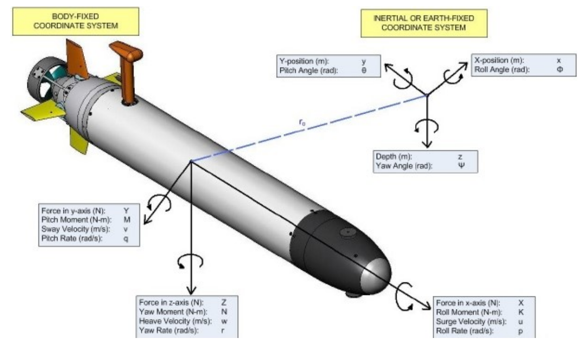

Fig. 1. Body frame and NED frame for vertical axis for a submarine vehicle

This lets $X = (w, q, z, \theta)$ to be the degrees of freedom of the AUV and d_s be the horizontal hydroplane angle. Eventually, the model is linearized around a steady depth and pitch and a constant speed u_0.

Using a different kind of payload can alter all the physical parameters of the system, from mass and inertia terms to hydrodynamic coefficients. Here, it is suggested to introduce a 10% uncertainty for all parameters of the AUV model around the nominal values computed by [1].

$$\dot{X} = A(p)X + B(p)\delta \tag{1}$$

C. Depth Control Strategy

The depth controller is a critical part of an AUV, especially for high speed, insofar as a bad design can cause a loss of the robot by crossing a critical depth. Performance goals can be adapted to control strategies and missions. However, a generic depth controller should cancel depth tracking error in minimum time, without imposing to much actuator consumption. Obviously, depth and pitch states should be stabilized. An optional requirement could be to minimize the overshoot during a depth transition. The suggested controller architecture is a full-state feedback regulator with an integral action on the depth to erase static error. Its transfer function is given by $K(s) = [k_{dz}, \quad k_{d\theta}, \quad k_{pz} + \frac{k_{iz}}{s}, \quad k_{p\theta}]$ with $k = (k_{dz}, k_{d\theta}, k_{pz}, k_{iz}, k_{p\theta})$ as the set of its tunable parameters and s the Laplace variable.

III. H_∞ Robust Control Problem

A. H_∞ Problem Formulation

H_∞ synthesis is a method to design controllers from frequency specifications. Its classical formulation involves the regulation scheme in the figure (2) where K is the the controller to compute and P is the plant to control, seen as Linear Time Invariant (LTI) systems. w represents the vector of inputs perturbations, z the regulated outputs, u is the control signal and y are the measured outputs.

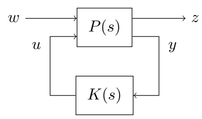

Fig. 2. Standard regulation scheme.

Let $F(P, K)$ be the Linear Fractional Transform of P and K. It can be described as a matrix of transfer functions that maps the inputs w to the outputs z, such as,

$$F(G, K) = \begin{pmatrix} T_{w\to z_1}(s) \\ \vdots \\ T_{w\to z_q}(s) \end{pmatrix}$$

where s is the Laplace variable and $T_{w\to z_i}(s) = (T_{w_1\to z_i}(s),$
$..., T_{w_n\to z_i}(s))$ is a row vector that maps w to z_i.

Practically, the goal of the H_∞ synthesis is to compute a controller that minimizes the maximal response of the outputs z from inputs w over the frequencies. The maximal responses can be quantified by the H_∞ norm. With the $||.||_\infty$ norm defined as $||T_{w\to z_i}||_\infty = \sup\limits_{\omega \neq 0} \frac{||z_i(j\omega)||_2}{||w(j\omega)||_2}$, the H_∞ synthesis can be defined as the following optimization problem:

The plant P is built from G the plant to control, augmented with weighting filters that amplify non-desired behaviors of the objective outputs. With $T_{w\to z_i}(s)$ the ith channel of G, and W an associated weighting filter, the corresponding transfer of the plant P will be $T_{w\to \tilde{z}_i}(j\omega) = W(j\omega)T_{w\to z_i}(j\omega)$. $\tilde{z}_i$ is the weighted objective output. W can be chosen as a frequency template. If $||WT_{w\to z}||_\infty \leq 1$, then W bounds the frequency response of $T_{w\to z}$.

A adequate choice for the problem formulation and the weightings W allows the H_∞ synthesis to be a powerful tool with multiple objectives such as disturbance rejection or the minimization of tracking error.

B. Solving the H_∞ synthesis problem

The solution of the H_∞ synthesis problem can be computed by solving Riccati equations as in [3] or by using Linear Matrix Inequalities as in [4]. However those methods fail to provide structured controllers that make LPV implementation difficult.

In this paper it is suggested to use an innovative optimization algorithm to provide a structured solution to the H_∞ synthesis problem with an guaranteed robustness to explicit model uncertainties. This method is fully described in [5]. It suggests to formulate the H_∞ problem as a minimax global optimization problem under non-convex constraints and to solve it with a branch and bound algorithm based on interval analysis.

First we translate the standard H_∞ problem into a minimax optimization problem. The structured controller K has the parameter gains $k \in \mathbb{R}^n$ and the model uncertain parameters are defined as:

$$p \in \{[p_{1_{min}} \quad p_{1_{max}}], ..., [p_{m_{min}} \quad p_{m_{max}}]\}.$$

The optimization goal is to find the appropriate controller gain values k that minimize the maximal values of the H_∞ norms set through uncertain parameters. As the H_∞ norm is also the maximal gain value through pulsation, the two maximum are merged in the expression given in equation (2):

$$\begin{cases} \min\limits_{k\in\mathbb{R}^n} (\max\limits_{i\in\{1,...,q\}, p\in P, \omega\in\mathbb{R}} (|T_{w\to z_i}(j\omega, p, k)|^2)) \\ \text{subject to } K(k) \text{ stabilizes } F(P, K) \end{cases} \quad (2)$$

IV. Application

A. Translation of control goals into H_∞ criteria

The requirements from Section II-C are now translated as H_∞ criteria. Depth tracking error and plane consumption minimization can be explicitly interpreted as the minimization of the H_∞ norm of depth and stern plane sensitivity, respectively $T_{r_z\to e_z}(k, p)$ and $T_{r_z\to d_s}(k, p)$. Complete static error cancellation can be reached by bounding depth sensitivity on low frequencies, forcing the use of the integral gain. It has been empirically observed that the overshoot is tightly correlated with the maximum value of the sensitivity, or its H_∞ norm. It is suggested to bound the depth sensitivity with a constant close to 0 dB.

One can notice that aforementioned methods for H_∞ synthesis implied to uses a stable inverse weighting functions. It is not the case with our synthesis method which gives more freedom to the designer. We take this opportunity to make criteria formal expression as simple as possible in order to make it faster to compute using the optimization algorithm. Hence, the depth sensitivity will be bounded by the weighting function $W_z(s) = \min(20s, 1.6)$ and the depth sensitivity by $W_z(s) = 1.6$. Eventually, the stability requirement is obtained by the internal stabilization of the closed-loop system.

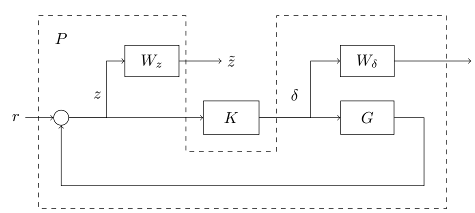

Fig. 3. Control scheme with weighting functions

The control scheme presented on Figure 3 shows the location of weighting functions and the regulated outputs. It follows the standard regulation scheme on Figure 2.

Robust internal stability can be validated by the computation of an analytic criterion such as the Lienard-Chipart Criterion from [6]. It is derived from the Routh-Hurwitz Criterion but it has a reduced computation complexity. [7]

shows how the validation of this criterion can be translated into a constraint satisfaction problem.

Lienard-Chipart stability criterion provides a set of coefficients built from a combination of coefficients of the characteristic polynomial of the system. Stability is guaranteed if all of them are strictly positive. A problem with this criterion when used with uncertain parameters is that the size of its formal expression increases dramatically with the characteristic polynomial which is the order of the closed-loop system. Let $\{T_{stab}(p,k)\}$ be the set of coefficients given by the Lienard-Chipart Criterion.

Eventually, the synthesis problem is summarized by equation(3):

$$\begin{cases} \min_{k}(\max_{p} ||W_z T_{r_z \to e_z}(k,p)||_\infty, ||W_\delta T_{r_z \to \delta}(k,p)||_\infty), \\ \text{such as } T_{stab}(p,k) > 0. \end{cases} \tag{3}$$

B. Problem Implementation

The main inputs of the synthesis algorithm are formal expressions of the criteria. Their complexity increases with the number of uncertain parameters, controller gains and order of the system which increase their evaluation computation time. Moreover, as illustrated in [8], this complexity also increases evaluation pessimism, dramatically slowing the convergence of the optimization algorithm. It is possible to attenuate this phenomenon by preserving factorized forms in the expression with the polynomial Horner form. We used the Symbolic Matlab Toolbox in order to compute formal expression faster with reliability. During the implementation of the specific control problem described in this work, the stability criteria evaluation appeared to be the most critical in term of computation time and convergence speed.

V. Results

Here, the control problem formulated in the previous section is solved by the algorithm presented in [5]. The starting box for the gain values is $[-70, 70]$ except for the integral gain, k_{i_z} which is $[-1, 1]$. Once gives an uncertainty of 10% for the model uncertain parameters. In this configuration, the control gains found by the optimizer are given in Table I. The H_∞ norm is included in the interval $[0.381\ 0.939]$ which was found in about 10 minutes on a standard computer.

TABLE I

Controller gains

Name	Gain value
k_{d_z}	-4.8331
k_{d_θ}	-9.7080
k_{p_z}	7.5631
k_{p_θ}	-58.7074
k_{i_z}	0.00114

The controller built by the presented method is valid and meets all specifications defined in Section II-C, and is guaranteed as optimal towards the criterion established in Section IV.

For an illustration, Figure 4 shows the sensitivity function of the depth reference error and the associated weighting function. We can see that the integral action is efficient for low frequencies and that the maximal gain was successfully contained by the weighting function. We can notice that the spread due to parameter's variations is condensed, meaning that the regulated system is robust for a considered model parameter error.

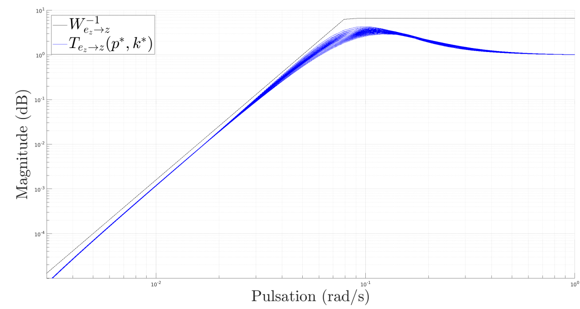

Fig. 4. Sensitivity of depth subject to depth perturbations in the closed loop system for a speed of $U = 1m/s$

VI. Conclusions

This work presented a formulation for the regulation problem of the submarine depth using H_∞ methods, taking into account parametric model uncertainties. It achieved to find an efficient controller using an innovative synthesis algorithm. The structured nature of the controller makes it implementable into a submarine systems. Further works could focus on increasing computation efficiency and problem formulation of the synthesis to tackle higher order problems.

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